Analytical Mechanics Final Exam.

1. A bead of mass m is constrained to slide along a thin, circular hoop of radius l. The hoop rotates with a constant angular velocity ω in a horizontal plane around point O on its rim, as illustrated in Figure 1.

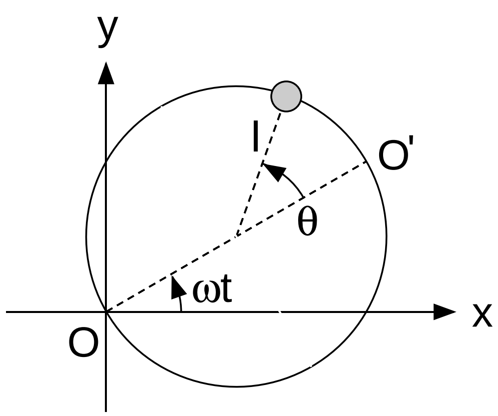

Figure 1: Bead moving along hoop

(a) Find the Lagrange equation of motion of the bead.
(b) Let O' be the point on the hoop rim diametrically opposite to point O. Show that the bead oscillates like a simple pendulum about point O'.

2. Let us investigate the transversal vibration of a beam. The beam is of length L and its one end is fixed on a wall, as illustrated in Figure 2. Force $f(t)$ is applied to the other end at time t. Let μ be the line density of the beam, E be its Young's module, and I be its geometrical moment of inertia. Let x be the distance from the wall and $u(x,t)$ be the traversal displacement at distance x and time t, as illustrated in the figure. Kinetic energy and bend potential energy of the beam are then described as follows, respectively:

$$T = \int_0^L \frac{1}{2}\mu \left(\frac{\partial u}{\partial t}\right)^2 dx,$$

$$U = \int_0^L \frac{1}{2}EI \left(\frac{\partial^2 u}{\partial x^2}\right)^2 dx.$$

Work done by the external force is described as

$$Work = f(t)u(L,t).$$

Compute the variation of action integral

$$\delta \int_{t_1}^{t_2} (T - U + Work)\, dt$$

and derive a differential equation that $u(x,t)$ must satisfy.

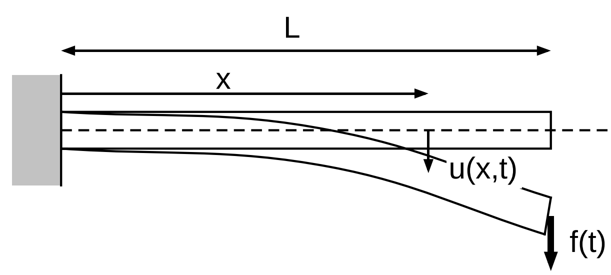

Figure 2: Transversal vibration of beam

transversal vibration	横振動
line density	線密度
Young's module	ヤング率
geometrical moment of inertia	断面二次モーメント
action integral	作用積分