**Analytical Mechanics Final Exam.**

**1**. Let us investigate the transversal vibration of a beam. The beam is of length $L$ and its one end is fixed on a wall, as illustrated in Figure 1. Force $f(t)$ is applied to the other end at time $t$. Let $\mu$ be the line density of the beam, $E$ be its Young's module, and $I$ be its geometrical moment of inertia. Let $x$ be the distance from the wall and $u(x,t)$ be the traversal displacement at distance $x$ and time $t$, as illustrated in the figure. Kinetic energy and bend potential energy of the beam are then described as follows, respectively:

$$T = \int_0^L \frac{1}{2}\mu \left(\frac{\partial u}{\partial t}\right)^2 \mathrm{d}x,$$

$$U = \int_0^L \frac{1}{2}EI \left(\frac{\partial^2 u}{\partial x^2}\right)^2 \mathrm{d}x.$$

Work done by the external force is described as

$$Work = f(t)u(L,t).$$

Compute the variation of action integral

$$\delta \int_{t_1}^{t_2} (T - U + Work)\, \mathrm{d}t$$

and derive a differential equation that $u(x,t)$ must satisfy.

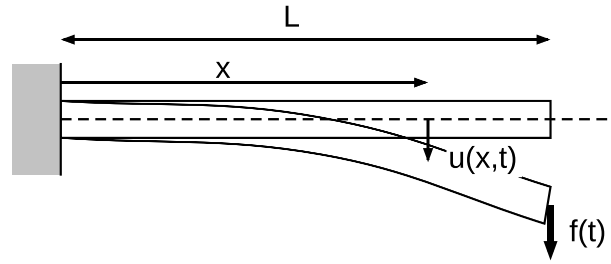


Figure 1: Transversal vibration of beam

transversal vibration
line density
Young's module
geometrical moment of inertia
action integral

**2**. A bead of mass $m$ moves along a inclined rigid bar. Friction between the bead and the bar is negligible. The bar rotates along a vertical plane and inclination of the bar is increasing at a constant rate $\omega$. Assuming that $\theta = 0$ at time $t = 0$, find the motion of the bead using Lagrangean formulation.

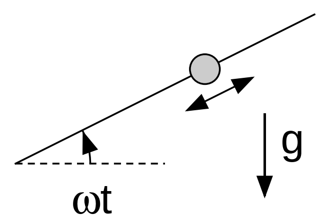


Figure 2: Bead moving along rotating bar