Analytical Mechanics Final Exam.

1. One simple pendulum of length l and mass m is suspended from a point on a circumference of a thin massless disk of radius a, as illustrated in Figure 1. The disk rotates with a constant angular velocity ω. Find the Lagrange equation of motion of the mass.

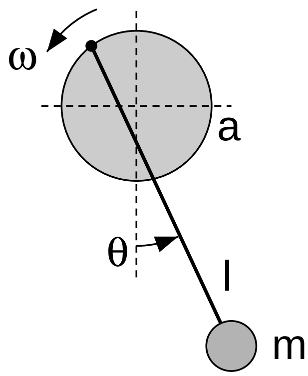

Figure 1: Pendulum attached on circulating point

2. Let us investigate the transversal vibration of a beam. The beam is of length L and its one end is fixed on a wall, as illustrated in Figure 2. Force $f(t)$ is applied to the other end at time t. Let μ be the line density of the beam, E be its Young's module, and I be its geometrical moment of inertia. Let x be the distance from the wall and $u(x,t)$ be the traversal displacement at distance x and time t, as illustrated in the figure. Kinetic energy and bend potential energy of the beam are then described as follows, respectively:

$$T = \int_0^L \frac{1}{2}\mu \left(\frac{\partial u}{\partial t}\right)^2 \,\mathrm{d}x,$$

$$U = \int_0^L \frac{1}{2}EI \left(\frac{\partial^2 u}{\partial x^2}\right)^2 \,\mathrm{d}x.$$

Work done by the external force is described as

$$W = f(t)u(L,t).$$

Assuming that μ, E, and I are constant, compute the variation of action integral

$$\delta \int_{t_1}^{t_2} (T - U + W)\,\mathrm{d}t$$

and derive a differential equation that $u(x,t)$ must satisfy.

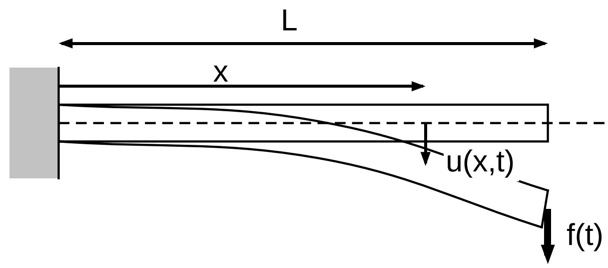

Figure 2: Transversal vibration of beam