Analytical Mechanics Report

due date: February 1 (Friday), 2008
should be submitted to Hirai's room (East 4F)

1. Let us formulate the one-dimensional viscoplastic deformation of a beam of length L and area of cross section A illustrated in Figure 1. Object deformation is described by Maxwell model, where E and η denote Young's modulus and viscous modulus of the object material. Let ρ be the line density of the object. Assume that E, η, ρ, and A are constant. The left end point of the object is fixed to space while force $f(t)$ is applied to the right end point of the object at time t. Let us describe the object deformation by five nodal points: P_0 through P_4. Derive a set of dynamic equations by applying a finite element approach. In addition, simulate the deformation of a viscoplastic beam. You may apply any numerical method for the integration of ordinary differential equations.

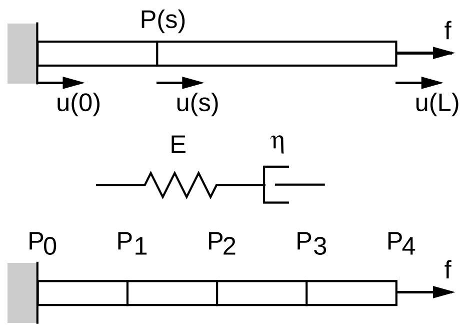

Figure 1: Maxwell object deformation

2. Let us bend a paper of length L and of uniform width on a table by decreasing the distance between two fingers pushing the both end of the paper. Assume that the bend is one-dimensional and investigate the cross section of the paper, as illustrated in Figure 2. Let s be the distance from the left end along the paper. Let $P(s)$ be a point on the paper specified by distance s. Let $\theta(s)$ be the angle from the horizon at point $P(s)$. Bend potential energy U is then formulated as

$$U = \int_0^L \frac{1}{2} R_f \left(\frac{\mathrm{d}\theta}{\mathrm{d}s} \right)^2 \mathrm{d}s,$$

where R_f denotes the bend rigidity of the paper. Assume that bend rigidity R_f is constant. Let $x(s)$ and $z(s)$ be coordinates at point $P(s)$, which are described as

$$x(s) = \int_0^s \cos\theta(u)\,\mathrm{d}u,$$
$$z(s) = \int_0^s \sin\theta(u)\,\mathrm{d}u.$$

Let ℓ be the distance between the two fingers. Assume that the gravitational potential energy is negligible.

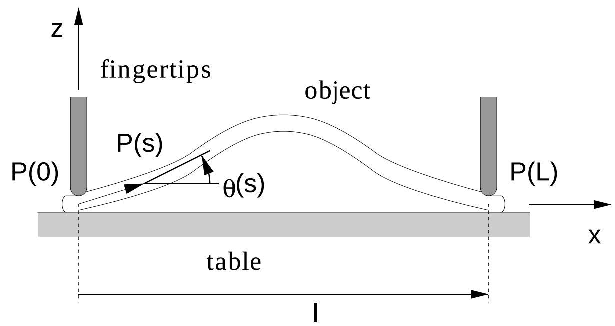

Figure 2: Bend of paper on table

The statically stable deformed shape of a paper can be computed by solving the following variational problem:

$$\begin{aligned}
\min \quad & U = \int_0^L \frac{1}{2} R_f \left(\frac{\mathrm{d}\theta}{\mathrm{d}s}\right)^2 \mathrm{d}s \\
\text{subject to} \quad & \theta(0) = 0, \quad \theta(L) = 0, \\
& x(L) = \int_0^L \cos\theta(s)\,\mathrm{d}s = \ell, \\
& z(L) = \int_0^L \sin\theta(s)\,\mathrm{d}s = 0.
\end{aligned}$$

Applying a finite element analysis to the above variational problem, compute the deformed shape of a paper.