Analytical Mechanics Report A

due date: February 2 (Monday), 2009
should be submitted to Hirai's room (East 4F)

1. A bead of mass m is sliding along a thin, circular hoop of radius l, as illustrated in Figure 1. The hoop rotates with a constant angular velocity ω in a horizontal plane around point O on its rim. Let C be the center of the circular hoop and P be the point on the hoop rim diametrically opposite to point O. Let θ be the angle between CP and a line connecting point C and the bead.

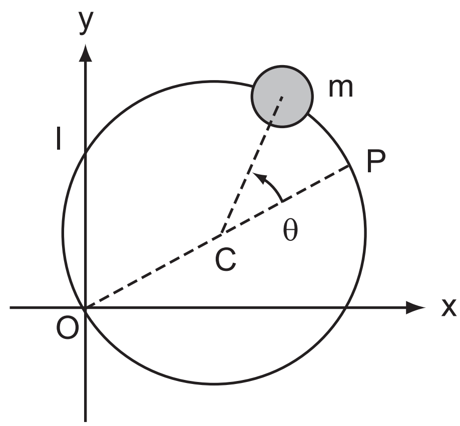

Figure 1: Bead moving along hoop

(a) Find the Lagrange equation of motion of the bead.
(b) Show that the bead oscillates like a simple pendulum about point P.

2. Simulate an indirect simultaneous positioning of an elastic 1D object illustrated in Figure 2. Two points P_1 and P_2 on the object must be guided to their desired location marked as crosses by controlling the position of both end points P_0 and P_3, as shown in the figure. Assume that the object shows viscoelastic deformation specified by Young's modulus E and viscous modulus c. Let ρ be the density of the object and A be its cross-sectional area. Apply the following the integral law based on the nearest relationship:

$$u_0(t) = K_I \int_0^t (u_1^* - u_1(\tau))\,\mathrm{d}\tau, \qquad u_3(t) = K_I \int_0^t (u_2^* - u_2(\tau))\,\mathrm{d}\tau.$$

where u_0 through u_3 be displacement of point P_0 through P_3 and u_1^* and u_2^* specify the desired location of two positioned points. Select an appropriate K_I to simulate the ISP process dynamically.

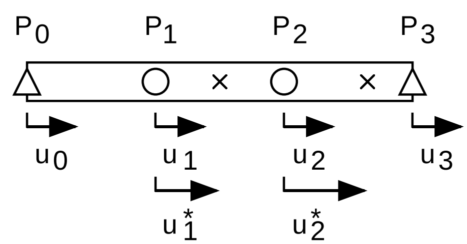

Figure 2: Indirect simultaneous positioning

Analytical Mechanics Report B

due date: February 2 (Monday), 2009
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1. A bead of mass m moves along a inclined rigid bar. Friction between the bead and the bar is negligible. The bar rotates along a vertical plane and inclination of the bar is increasing at a constant angular velocity ω, as illustrated in Figure 1. Assuming that $\theta = 0$ at time $t = 0$, find the motion of the bead using Lagrangian formulation.

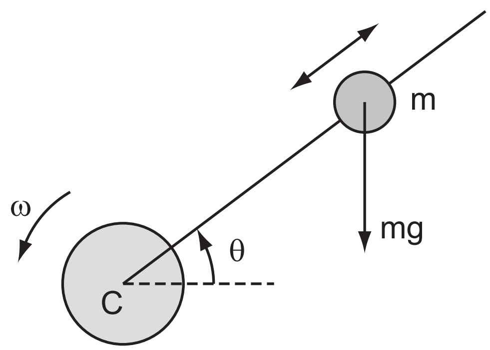

Figure 1: Bead moving along rotating bar

2. Simulate the 2D viscoplastic deformation of a rectangular object. Let $\lambda^{\rm ela}$ and $\mu^{\rm ela}$ specify the elasticity of the object, $\lambda^{\rm vis}$ and $\mu^{\rm vis}$ determine its viscosity, and ρ be its density. Assume that one edge of the rectangular object is fixed to a rigid wall and an external force is applied to a point on its opposite edge for a while.

Analytical Mechanics Report C

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1. A simple pendulum of length l and mass m is suspended from point C, as illustrated in Figure 1. Supporting point C is moving along x-axis and its position at time t is given by $x = d(t)$. Let θ be the angle of the pendulum. Derive the Lagrangian of this system and its Lagrange equation of motion.

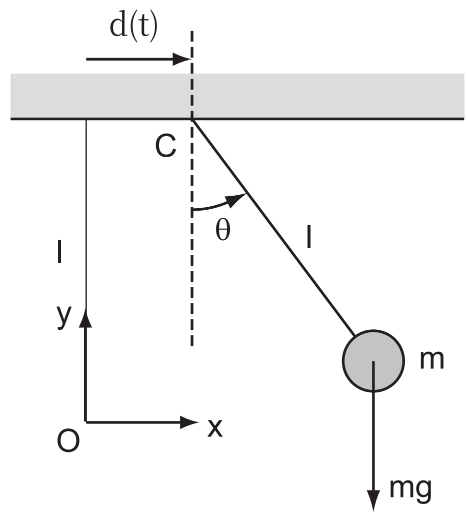

Figure 1: Pendulum attached on moving point

2. Let us investigate the dynamics of a planar 2-DOF link mechanism illustrated in Figure 1. Let l_1 and l_2 be the length of the two links. The position of the gravity center of each link is specified by l_{c1} and l_{c2}, respectively. Let m_1 and m_2 be mass of the two links and I_1 and I_2 be their inertia around the gravity center. Motion equations of the mechanism are then formulated as

$$H_{11}\ddot{\theta}_1 + H_{12}\ddot{\theta}_2 - h\dot{\theta}_2^2 - 2h\dot{\theta}_1\dot{\theta}_2 + G_1 + G_{12} = \tau_1,$$
$$H_{22}\ddot{\theta}_2 + H_{12}\ddot{\theta}_1 + h\dot{\theta}_1^2 + G_{12} = \tau_2,$$

where

$$H_{11} = m_1 l_{c1}^2 + I_1 + m_2(l_1^2 + l_{c2}^2 + 2 l_1 l_{c2}\cos\theta_2) + I_2,$$
$$H_{12} = m_2(l_{c2}^2 + l_1 l_{c2}\cos\theta_2) + I_2,$$
$$H_{22} = m_2 l_{c2}^2 + I_2,$$
$$h = m_2 l_1 l_{c2}\sin\theta_2,$$
$$G_1 = (m_1 l_{c1} + m_2 l_1) g \cos\theta_1,$$
$$G_{12} = m_2 l_{c2} g \cos(\theta_1 + \theta_2).$$

Derive the above equations using Lagrangian formulation. Apply the following PID control law to investigate the behavior of the system numerically.

$$\tau_1(t) = -K_{\mathrm{P}}(\theta_1 - \theta_1^{\mathrm{d}}) - K_{\mathrm{D}}\dot{\theta}_1 - K_I \int_0^t (\theta_1(\tau) - \theta_1^{\mathrm{d}})\,\mathrm{d}\tau,$$
$$\tau_2(t) = -K_{\mathrm{P}}(\theta_2 - \theta_2^{\mathrm{d}}) - K_{\mathrm{D}}\dot{\theta}_2 - K_I \int_0^t (\theta_2(\tau) - \theta_2^{\mathrm{d}})\,\mathrm{d}\tau,$$

where θ_1^{d} and θ_2^{d} are desired values of angles θ_1 and θ_2.

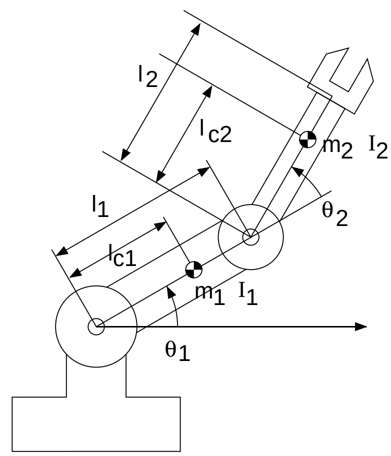

Figure 2: Link mechanism involving two joints

Analytical Mechanics Report D

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1. A particle of mass m is free to slide along a smooth cycloidal trough under gravity, acting in the negative direction along the y-axis. The surface of the trough is given by a parametric equations

$$x = a(\theta - \sin\theta),$$
$$y = a(1 - \cos\theta),$$

where a is a positive constant. Find the Lagrangian of the particle and derive the equation of motion of the particle. Note that a cycloid is a curve drawn by a point on a circle of radius a rolling along a flat surface.

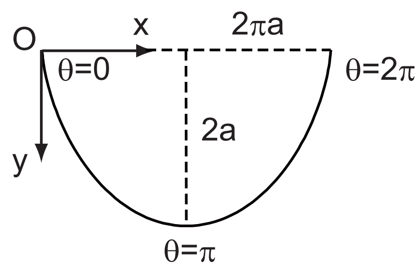

Figure 1: Cycloidal trough

2. Figure 2 shows a closed link mechanism. One end of link 1 is attached to space via rotational joint 1, which is driven by a motor fixed to space. The other end of the link is connected to link 2 via rotational joint 2. Joint 2 is free, implying that link 1 and 2 can rotate around this joint freely. One end of link 3 is attached to space via rotational joint 3, which is driven by a motor fixed to space. The other end of the link is connected to link 4 via rotational joint 4. Joint 4 is free, implying that link 3 and 4 can rotate around this joint freely. In addition, the other end of link 2 and that of link 4 is connected via free joint 5. Assume that all axis of rotational joints are parallel one another. As shown in the figure, let θ_i be the angle of rotation of the i-th joint and ω_i be its angular velocity. Apply the following PID control law to investigate the behavior of the system numerically.

$$\tau_1(t) = -K_{\rm P}(\theta_1 - \theta_1^{\rm d}) - K_{\rm D}\dot{\theta}_1 - K_I \int_0^t (\theta_1(\tau) - \theta_1^{\rm d})\,{\rm d}\tau,$$
$$\tau_3(t) = -K_{\rm P}(\theta_3 - \theta_3^{\rm d}) - K_{\rm D}\dot{\theta}_3 - K_I \int_0^t (\theta_3(\tau) - \theta_3^{\rm d})\,{\rm d}\tau$$

where $\theta_1^{\rm d}$ and $\theta_3^{\rm d}$ are desired values of angles θ_1 and θ_3.

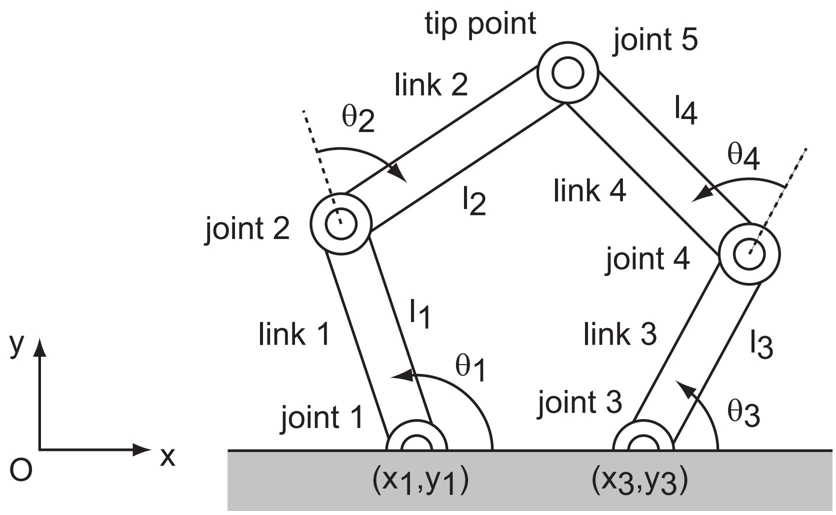

Figure 2: Closed link mechanism

Analytical Mechanics Report E

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1. Let us bend a paper of length L and of uniform width on a table by decreasing the distance between two fingers pushing the both end of the paper. Assume that the bend is one-dimensional and investigate the cross section of the paper, as illustrated in Figure 1. Let s be the distance from the left end along the paper. Let $P(s)$ be a point on the paper specified by distance s. Let $\theta(s)$ be the angle from the horizon at point $P(s)$. Bend potential energy U is then formulated as

$$U = \int_0^L \frac{1}{2} R_f \left(\frac{\mathrm{d}\theta}{\mathrm{d}s} \right)^2 \mathrm{d}s,$$

where R_f denotes the bend rigidity of the paper. Assume that bend rigidity R_f is constant. Let $x(s)$ and $z(s)$ be coordinates at point $P(s)$, which are described as

$$\begin{aligned} x(s) &= \int_0^s \cos\theta(u)\,\mathrm{d}u, \\ z(s) &= \int_0^s \sin\theta(u)\,\mathrm{d}u. \end{aligned}$$

Let ℓ be the distance between the two fingers. Assume that the gravitational potential energy is negligible. Then, we can derive the deformed shape of a paper by solving the following variational problem:

$$\begin{aligned} \min \quad & U = \int_0^L \frac{1}{2} R_f \left(\frac{\mathrm{d}\theta}{\mathrm{d}s} \right)^2 \mathrm{d}s \\ \text{subject to} \quad & \theta(0) = 0, \quad \theta(L) = 0, \\ & x(L) = \int_0^L \cos\theta(s)\,\mathrm{d}s = \ell, \\ & z(L) = \int_0^L \sin\theta(s)\,\mathrm{d}s = 0. \end{aligned}$$

Applying the Euler-Lagrange equation in variation, show that the above variation problem is equivalent to the following differential equation:

$$R_f \frac{\mathrm{d}^2\theta}{\mathrm{d}s^2} + \lambda_x \sin\theta - \lambda_y \cos\theta = 0,$$

where λ_x and λ_y are Lagrange multipliers.

2. Simulate the 2D viscoplastic deformation of a rectangular object. Let λ^{ela} and μ^{ela} specify the elasticity of the object, λ^{vis} and μ^{vis} determine its viscosity, and ρ be its density. Assume that one edge of the rectangular object is fixed to a rigid wall and an external force is applied to a point on its opposite edge for a while.

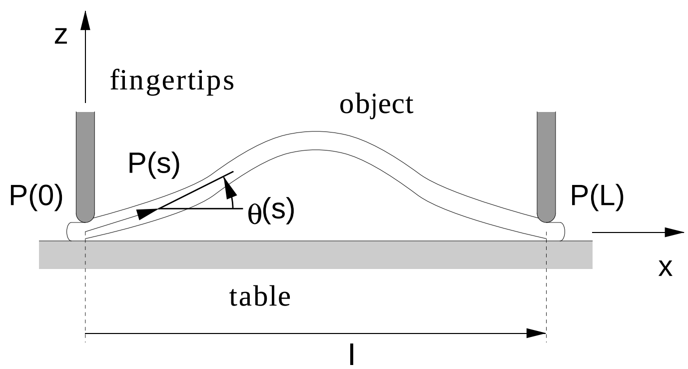

Figure 1: Bend of paper on table

Analytical Mechanics Report F

due date: February 2 (Monday), 2009
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1. A simple pendulum of length l and mass m is suspended from a point on a circumference of a thin massless disk of radius a, as illustrated in Figure 1. The disk rotates around its center C with a constant angular velocity ω. Find the Lagrange equation of motion of the mass.

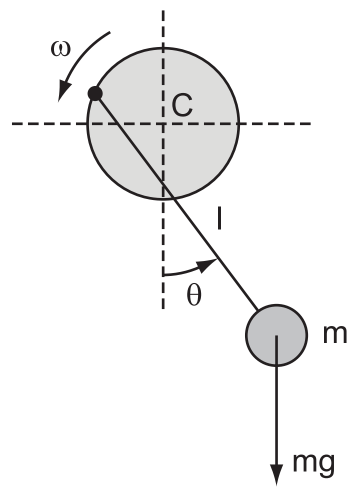

Figure 1: Pendulum attached on circulating point

2. Simulate an indirect simultaneous positioning of an elastic 1D object illustrated in Figure 2. Two points P_1 and P_2 on the object must be guided to their desired location marked as crosses by controlling the position of both end points P_0 and P_3, as shown in the figure. Assume that the object shows viscoelastic deformation specified by Young's modulus E and viscous modulus c. Let ρ be the density of the object and A be its cross-sectional area. Apply the following the integral law based on the nearest relationship:

$$u_0(t) = K_I \int_0^t (u_1^* - u_1(\tau))\, \mathrm{d}\tau, \qquad u_3(t) = K_I \int_0^t (u_2^* - u_2(\tau))\, \mathrm{d}\tau.$$

where u_0 through u_3 be displacement of point P_0 through P_3 and u_1^* and u_2^* specify the desired location of two positioned points. Select an appropriate K_I to simulate the ISP process dynamically.

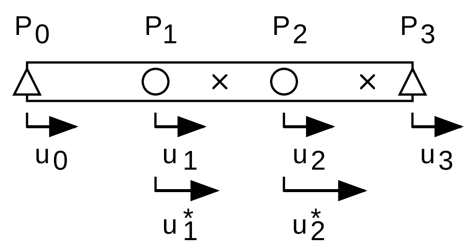

Figure 2: Indirect simultaneous positioning

Analytical Mechanics Report G

due date: February 2 (Monday), 2009
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1. A pendulum of rod length l and mass m is suspended from point C, as illustrated in Figure 1. The rod length varies according to time and the length at time t is given by $l(t) = l_0 + A\sin\omega t$, where l_0 is a positive constant, ω denotes a constant angular frequency, and A denotes a constant amplitude that satisfies $-l_0 < A < l_0$. Let x and y be coordinates of the mass. Derive the Lagrange equation of motion of the system and solve the derived equations numerically.

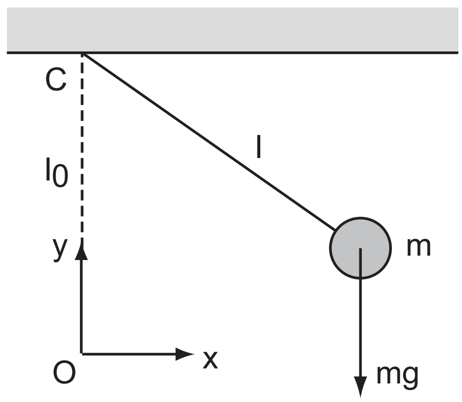

Figure 1: Pendulum with variable rod

2. Figure 2 shows a closed link mechanism. One end of link 1 is attached to space via rotational joint 1, which is driven by a motor fixed to space. The other end of the link is connected to link 2 via rotational joint 2. Joint 2 is free, implying that link 1 and 2 can rotate around this joint freely. One end of link 3 is attached to space via rotational joint 3, which is driven by a motor fixed to space. The other end of the link is connected to link 4 via rotational joint 4. Joint 4 is free, implying that link 3 and 4 can rotate around this joint freely. In addition, the other end of link 2 and that of link 4 is connected via free joint 5. Assume that all axis of rotational joints are parallel one another. As shown in the figure, let θ_i be the angle of rotation of the i-th joint and ω_i be its angular velocity. Apply the following PID control law to investigate the behavior of the system numerically.

$$\tau_1(t) = -K_{\mathrm{P}}(\theta_1 - \theta_1^{\mathrm{d}}) - K_{\mathrm{D}}\dot{\theta}_1 - K_I \int_0^t (\theta_1(\tau) - \theta_1^{\mathrm{d}})\,\mathrm{d}\tau,$$

$$\tau_3(t) = -K_{\mathrm{P}}(\theta_3 - \theta_3^{\mathrm{d}}) - K_{\mathrm{D}}\dot{\theta}_3 - K_I \int_0^t (\theta_3(\tau) - \theta_3^{\mathrm{d}})\,\mathrm{d}\tau$$

where θ_1^{d} and θ_3^{d} are desired values of angles θ_1 and θ_3.

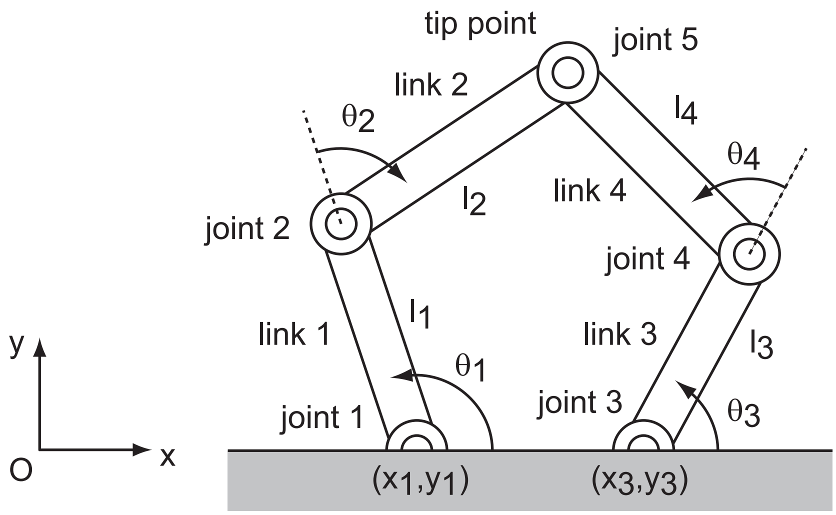

Figure 2: Closed link mechanism

Analytical Mechanics Report H

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1. Two blocks of equal mass M are connected by a bendable but inextensible cord of length l. One block is placed on a smooth horizontal table. The other block hangs over the edge of the table. Find the Lagrangian of the system and the acceleration of the blocks and the cord assuming (1) the cord is light, say, its mass is negligible and (2) the cord is heavy, say, its mass is given by m.

2. Let us investigate the dynamics of a planar 2-DOF link mechanism illustrated in Figure 1. Let l_1 and l_2 be the length of the two links. The position of the gravity center of each link is specified by l_{c1} and l_{c2}, respectively. Let m_1 and m_2 be mass of the two links and I_1 and I_2 be their inertia around the gravity center. Motion equations of the mechanism are then formulated as

$$H_{11}\ddot{\theta}_1 + H_{12}\ddot{\theta}_2 - h\dot{\theta}_2^2 - 2h\dot{\theta}_1\dot{\theta}_2 + G_1 + G_{12} = \tau_1,$$
$$H_{22}\ddot{\theta}_2 + H_{12}\ddot{\theta}_1 + h\dot{\theta}_1^2 + G_{12} = \tau_2,$$

where

$$H_{11} = m_1 l_{c1}^2 + I_1 + m_2(l_1^2 + l_{c2}^2 + 2l_1 l_{c2}\cos\theta_2) + I_2,$$
$$H_{12} = m_2(l_{c2}^2 + l_1 l_{c2}\cos\theta_2) + I_2,$$
$$H_{22} = m_2 l_{c2}^2 + I_2,$$
$$h = m_2 l_1 l_{c2}\sin\theta_2,$$
$$G_1 = (m_1 l_{c1} + m_2 l_1) g\cos\theta_1,$$
$$G_{12} = m_2 l_{c2} g\cos(\theta_1 + \theta_2).$$

Derive the above equations using Lagrangian formulation. Apply the following PID control law to investigate the behavior of the system numerically.

$$\tau_1(t) = -K_{\mathrm{P}}(\theta_1 - \theta_1^{\mathrm{d}}) - K_{\mathrm{D}}\dot{\theta}_1 - K_I\int_0^t (\theta_1(\tau) - \theta_1^{\mathrm{d}})\,\mathrm{d}\tau,$$
$$\tau_2(t) = -K_{\mathrm{P}}(\theta_2 - \theta_2^{\mathrm{d}}) - K_{\mathrm{D}}\dot{\theta}_2 - K_I\int_0^t (\theta_2(\tau) - \theta_2^{\mathrm{d}})\,\mathrm{d}\tau,$$

where θ_1^{d} and θ_2^{d} are desired values of angles θ_1 and θ_2.

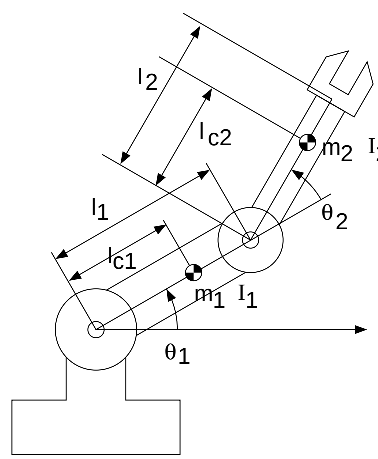

Figure 1: Link mechanism involving two joints

Analytical Mechanics Report I

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1. Recall that the motion of a particle of mass m and electric charge q moving with velocity $\boldsymbol{v}$ in a static magnetic field $\boldsymbol{B}$ is formulated by a differential equation of motion

$$m\ddot{\boldsymbol{r}} = q(\boldsymbol{v} \times \boldsymbol{B}),$$

where vector $\boldsymbol{r}$ specifies the position of the particle. Assume that vector $\boldsymbol{B}$ is given by the rotation of vector $\boldsymbol{A}$, that is,

$$\boldsymbol{B} = \mathrm{rot}\boldsymbol{A} = \nabla \times \boldsymbol{A} = \begin{bmatrix} \dfrac{\partial A_z}{\partial y} - \dfrac{\partial A_y}{\partial z} \\ \dfrac{\partial A_x}{\partial z} - \dfrac{\partial A_z}{\partial x} \\ \dfrac{\partial A_y}{\partial x} - \dfrac{\partial A_x}{\partial y} \end{bmatrix}.$$

Show that the Lagrangian function

$$L = \frac{1}{2}mv^2 + q\boldsymbol{v} \cdot \boldsymbol{A}$$

yields the above equation of motion correctly.

2. Simulate an indirect simultaneous positioning of an elastic 2D object illustrated in Figure 1. Three points marked as circles on the object must be guided to their desired location marked as crosses by controlling the position of three manipulated points marked as triangles, as shown in the figure. You may assume that the control process is static. Introduce the discrepancy between physical parameters in control law and actual ones to discuss the robustness of the control law.

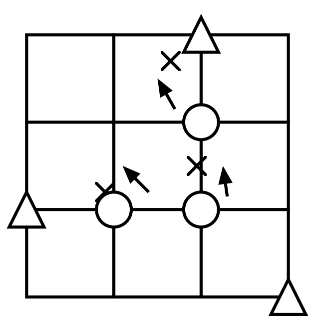

Figure 1: Indirect simultaneous positioning in 2D space